COUPLED-MODE ANALYSIS FOR CHARACTERISTIC IMPEDANCES OF COUPLED MICROSTRIP LINES ON FERRITE SUBSTRATES

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1. Introduction

A coupled-mode formulation for characteristic impedances of coupled microstrip lines on a magnetized ferrite substrate is presented. The formulation is an extension of the coupled-mode theory for microstrip lines on an isotropic substrate [1].

2. Formulation

We consider coupled microstrip lines on a ferrite substrate magnetized in the x direction as shown in Fig. 1. The permeability tensor of the ferrite is given as:

$$[\bar{\bar{\mu}}] = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & -i\kappa \\ 0 & i\kappa & \mu \end{bmatrix}, \quad \text{with} \quad \mu = 1 - \frac{\gamma^2 H_i M_i}{\omega^2 - (\gamma H_i)^2} \quad \text{and} \quad \kappa = \frac{\gamma M_i \omega}{\omega^2 - (\gamma H_i)^2} \tag{1}$$

where H_i is the internal dc magnetic field, M_i is the saturation magnetization, and γ is the gyromagnetic ratio. The coupled-mode equations for the microstrip lines are derived as follows:

$$\frac{d}{dz}\boldsymbol{a} = -i[\boldsymbol{C}]\boldsymbol{a}, \ \text{with}\, \boldsymbol{a} = [a_1\ a_2]^T, \ [\boldsymbol{C}] = [\boldsymbol{M}]^{-1}[\boldsymbol{K}], \ K_{\nu\mu} = \beta_\nu^{(0)} M_{\nu\mu} + Q_{\nu\mu}, \ M_{\nu\mu} = \frac{1}{2}(N_{\nu\mu} + N_{\mu\nu}) \tag{2}$$

$$N_{\nu\mu} = \frac{1}{2}\int_S [\boldsymbol{e}_\nu(x,y) \times \boldsymbol{h}_\mu(x,y)] \cdot \hat{\boldsymbol{z}}\,dxdy, \ Q_{\nu\mu} = -\frac{i}{4}\int_{l_\mu} [e_{\nu,x}(x,h_1) j_{\mu,x}(x) - e_{\nu,z}(x,h_1) j_{\mu,z}(x)]dx, \tag{3}$$

where $\boldsymbol{e}_\nu(x,y)$, $\boldsymbol{h}_\nu(x,y)$ $\boldsymbol{j}_\nu(x,y)$ and $\beta_\nu^{(0)}$ $(\nu = 1,2)$ are eigenmode functions for the fields, current and the propagation constant propagating in the $+z$ direction along ν-th lines in isolation, and $a_\nu(z)$ is an unknown amplitude function. The solutions determine the forward and backward propagation constants $\beta_m^{(\pm)}$ of the coupled mode m and the modal amplitudes $a_{\nu m}^{(\pm)}$ of the current on the ν-th line. The characteristic mode impedance $Z_{c,\nu m}^{(\pm)}$ of the ν-th line for mode m is defined using the eigencurrents:

$$V_{\nu m}^{(\pm)} = Z_{c,\nu m}^{(\pm)} I_{\nu m}^{(\pm)}, \quad I_{\nu m}^{(\pm)} = a_{\nu m}^{(\pm)} e^{-i\beta_m^{(\pm)} z} \int_{x_\nu - w_\nu}^{x_\nu + w_\nu} j_{\nu,z}(x)dx, \quad (\nu, m = 1,2) \tag{4}$$

where $V_{\nu m}^{(\pm)}$ represents the eigenvoltage. $I_{\nu m}^{(\pm)}$ and $V_{\nu m}^{(\pm)}$ are related to the total power $P_m^{(\pm)}$ carried by mode m in the z direction as follows:

$$\frac{1}{2}\sum_{\nu=1}^{N} V_{\nu m}^{(\pm)} I_{\nu m'}^{(\pm)*} = P_m^{(\pm)} \delta_{mm'}, \ P_m^{(\pm)} = \frac{1}{2}\int_S (\boldsymbol{E}_m^{(\pm)} \times \boldsymbol{H}_m^{(\pm)*}) \cdot \hat{\boldsymbol{z}}\,dxdy = \sum_{\nu=1}^{N}\left(\sum_{\mu=1}^{N} a_{\nu m}^{(\pm)} a_{\mu m}^{(\pm)} N_{\nu\mu}^{(\pm)}\right) \tag{5}$$

where $\delta_{mm'}$ is Kronecker's delta, $\boldsymbol{E}_m^{(\pm)}$ and $\boldsymbol{H}_m^{(\pm)}$ are the total electric and magnetic fields for mode m. Substituting Eqs. (4) into Eq. (5), the expression of $Z_{c,\nu m}^{(\pm)}$ in terms of the solutions to the coupled-mode equation (2) is derived.

3. Numerical results

Figure 1 shows the characteristic mode impedances as functions of frequency, where h_1=1.0mm, w=0.5mm, d=0.5mm, ε_r=12.7, H_i=102555A/m and M_i=170925A/m.

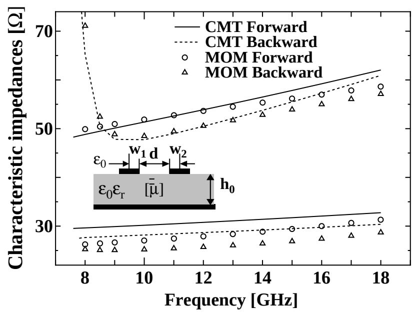

Fig. 1 Characteristic mode impedances of two identical microstrip lines.

References

[1] M. Matsunaga, M. Katayama, and K. Yasumoto, "Coupled-mode analysis of line parameters of coupled microstrip lines," *Progress in Electromagnetic Research*, vol.PIER-24, pp.1–18, 1999.