Experimental Determination of the Dependence of Tropospheric Pathlength Variation on Airmass

M.A. Holdaway
National Radio Astronomy Observatory
Socorro, NM 87801

Masato Ishiguro
Nobeyama Radio Observatory
Minamimaki, Minamisaku, Nagano
384-13, Japan

March 24, 1995

Abstract

The rms interferometric phase, presumed due to inhomogeneously distributed water vapor in the troposphere, is shown to vary as the square root of the airmass on baselines of a few hundreds of meters, matching the prediction of Treuhaft and Lanyi (1987). There is tentative evidence that the rms phase varies more steeply with airmass on baselines of about 700 m.

Introduction

Currently, the NRO, the CFA, and the NRAO are operating site testing interferometers at several locations in northern Chile and on Mauna Kea (see, for example, Ishiguro *et al.*, 1990. These site testing interferometers measure the tropospheric phase on a single baseline at about 1 cm wavelength by observing strong geostationary satellites. Since the elevation angle of the geostationary satellites will be different at different sites, a comparison of the phase stability on the different sites requires that the phase errors be corrected to a common elevation. This requires an understanding of the variation of the rms phase with elevation, or airmass (defined as $1/\sin(EL)$). Furthermore, estimates of the effectiveness of various phase correction schemes over the sky will require an understanding of the variation of rms phase with elevation.

Treuhaft and Lanyi (1987) predict that for baselines which are short compared to the scale height of the turbulence (taken to be the thickness of the tropospheric layer of turbulent water vapor), the rms phase fluctuations will vary as the square root of airmass, while for baselines which are long compared to the scale height of the turbulence, the rms phase fluctuations will vary proportionally to the airmass. Millimeter wavelength phase stability analysis has assumed the rms phase varies as the square root of the airmass (Masson, 1994) and as the airmass (Holdaway, 1991). We present observational results which indicate that the rms phase fluctuations on several hundred meter baselines vary as the square root of airmass.

Source (1950)	Flux* (mJy)	Elevation (degrees)	Airmass
	Subarray A		
0400+258	720	81	1.01
0458+000	2456	54	1.24
0414-172	1423	37	1.66
	Subarray B		
0414-172	1423	37	1.66
0454-226	1238	32	1.89
0451-282	1770	27	2.20
	Subarray C		
0451-282	1770	27	2.20
0534-340	842	21	2.79
0422-380	1700	18	3.24

* The flux of 0414-172 was chosen to be 1423 mJy.

Table 1: Sources observed for the determination of rms phase with airmass.

Observations

Observations of seven bright calibrators (two were observed twice) were carried out on March 9 from 4:00 to 5:00 LST. The sources are listed in Table 1. The sensitivity of the X Band (8 GHz) system makes it the optimal VLA frequency for detecting tropospheric path length fluctuations, giving a thermal rms of less than a degree per visibility (10 s averaging, 50 MHz bandwidth, 1 Jy source). The observations were taken in D array with a maximum baseline of about 1000 m. With a wind velocity of 10 m/s, the crossing time of the array is 100 s, or about 12 crossing times in a 20 minute integration, which is enough to approximate the ensemble average required for determining the phase structure function. Since the phase structure function changes on time scales of tens of minutes to hours, we broke the VLA into three subarrays, each containing three antennas from each arm, and observed each of the nine sources for about 20 minutes. Sources were observed while transiting so the elevation would not change and the azimuth would be nearly the same for all sources, ie, due south. Some of the antennas on the northern arm were shadowed during observations of the sources at $\delta = -28^\circ$ and below.

Reduction

The uncalibrated data were split into separate IFs and exported to SDE where we flagged bad data points, subtracted the average phase and determined the rms phase on each baseline, and fit a power law to the rms phase fluctuations as a function of the *zenith* baseline length ρ

$$\sigma_{\phi}(\rho) = A\rho^{\alpha}, \tag{1}$$

ignoring 3 σ outliers in the fitting. This expression is the square root of the phase structure function, and when we refer to the phase structure function below, we really mean the square root of the phase structure function.

The sources 0414-172 and 0451-282 were observed twice, at different times in the run and by different subarrays, to check for any systematic variation in the phase structure function during the observations. The third source observed by each of the subarrays was quite different from the other data, showing rms phase fluctuations only about half as large as expected from the other measurements, implying that the statistical properties of the atmosphere had changed. Hence, we have dropped the third source from each subarray, leaving measurements of the phase structure function at six different elevations ranging from 21° to 81°.

As an example, the rms phase fluctuations as a function of baseline and the fit power law for the source 0454-226 (IF 2) are shown in Figure 1.

Analysis

We found that the amplitude A and exponent α of the phase structure function are coupled in the least square fits. The phase structure functions determined by the two IFs on a given source should be similar since the frequencies are nearly identical and the baselines are identical. While the rms phases in the two IFs were usually quite similar, we sometimes found that the power law fit to the two IF's data gave amplitudes and exponents which were different by as much as 30%, and the fit to one IF was also a good fit to the other IF's data. Because an increased amplitude is compensated for by a decreased exponent in these cases (or vice versa), when the two IF's the spatial structure functions are evaluated at some representative baseline, they yield rms phases which agree very well. Hence, we cannot draw any firm conclusions from any variation of the *phase structure function's amplitude and exponent with airmass*, but we will draw conclusions from the rms phase on representative baselines.

The power law fits to the rms phase as a function of zenith baseline length were used to estimate the rms phase on 350 m baselines (a typical baseline length for each of the three subarrays of the D array). A second power law fit was then made for the 350 m baseline rms phase as a function of airmass m:

$$\sigma_{\phi}(m, \rho = 350m) = Bm^{\beta}, \tag{2}$$

yielding $\beta = 0.52$, consistent with the phase fluctuations varying as the square root of airmass. The data and the least squares fit are shown in Figure 2. The two data points at each airmass are for the two IFs, and give an estimate of the thermal rms. For low airmass, the data show significant scatter from the best fit line. This may be due to large scale inhomogeneities in the atmosphere. After this hypothesis, the interferometer would be looking through more such inhomogeneities in the atmosphere at the higher airmass observations, so their effects average down, possibly explaining the better agreement with the least squares power law fit. If the high data points at 1.24 airmass are neglected in the fit, the power law power is 0.64, still consistent with the theoretical value of 0.5.

Treuhaft and Lanyi (1987) also predict that the rms phase on baselines which are long compared with the tropospheric scale height will vary as the airmass ($\beta = 1$), rather than as the square root of the airmass ($\beta = 0.5$). To look for a variation in the airmass dependence with baseline length in our data, we restricted the power law fit to the phase structure function to the ranges 100-400 m, 175-700 m, and 350-1000 m, calculated the rms phase from the structure function fit at the representative baselines of 200 m, 350 m, and 700 m, and performed a power law fit to the rms phase as a function of airmass for these three different baselines. The fit power law powers are 0.53, 0.52, and 0.60 respectively. If one includes all data points in the 700 m fit, the rms phase varies as airmass raised to the 0.60 power, which is not significantly different from the theoretical 0.50. However, if the data points at the two low airmasses are not included in the fit, the rms phase at the remaining four airmasses varies as the airmass raised to the 1.1 power, which would indicate the scale height of the turbulent water vapor is less than 700 m. We might expect some deviation from the power law curve at low airmasses due to large scale atmospheric inhomogeneities. Higher airmass observations have a line of sight through the troposphere which cuts through several such inhomogeneities, averaging out their effect. Sramek (1989) found statistical evidence that the phase structure function at the VLA turns over on baselines of about 1000 m.

We have also estimated the value of β from a global fit approach to the data in which the rms phase fluctuations as a function of zenith baseline and airmass for all of the sources was assumed to be

$$\sigma_\phi(m, \rho) = A(m)^\beta \rho^\alpha. \tag{3}$$

$\chi^2(A, \alpha, \beta)$ has a fairly broad minimum (primarilly due to actual variations of the statistical properties of the atmosphere with time and elevation rather than to thermal noise), and a wide range of values of the parameters (A, α, β) gave nearly the same χ^2. The (A, α, β) grid points which gave the lowest χ^2 were averaged, leading to $\beta = 0.70 \pm 0.19$ when all baselines were included, $\beta = 0.83 \pm 0.24$ when just baselines greater than 400 m were included, and $\beta = 0.59 \pm 0.18$ when only baselines shorter than 400 m were included.

Discussion

The rms phase on short baselines (~400 m or less) appears to vary like the square root of the airmass. There is tentative evidence that the rms phase on longer baselines (~700 m) varies more steeply with airmass. The measurements are limited by the statistical properties of the atmosphere changing with time and changing with position in the atmosphere. Future observations might address this problem by time sharing between the sources at the different elevations every few minutes over an hour or two of observations so that temporal drifts of the statistical properties of the atmosphere would affect sources at all elevations.

Because the LMSA's maximum baselines are 2000 m and the MMA's longest baselines are 3000 m, it would seem to be relevant to perform test observations of the rms phase verses airmass on 3000 m baselines to verify the dependence of phase with airmass on these longer baselines. However, the phase correction schemes envisioned will "chop off" the structure function at some effective calibration baseline resulting in residual phase errors of $D_\phi(vt/2+d)$ (Holdaway 1992, 1995), and measurements of the phase dependence with airmass on such long baselines is probably not relevant. However, analysis of radiometric opacity fluctuations on the South Baldy site (Holdaway, 1991) indicates that the phase structure function has a turnover at a few hundred meters, implying the layer of turbulent water vapor is only a few hundred meters thick. If such a phase structure function is confirmed by the ongoing site testing efforts on Mauna Kea and Chile, each with 300 m baselines, and if the effective calibration baseline is greater than this height, then we must consider that the rms phase may vary as the airmass rather than as the square root of the airmass. (Current estimates of the effective calibration baseline for fast switching are about 50 m for Mauna Kea. The effective calibration baseline should be smaller for radiometric phase calibration and "split array" calibration.)

We must conclude with a final caveat: these observations are only for one hour at the VLA site. It is expected that the general trends of rms phase with airmass on short baselines should hold at other times, but the specific nature of the rms phase/airmass relationship with baseline would be expected to change with the weather, and to change with the site.

References

1. Holdaway, M. 1991, "A Millimeter Wavelength Phase Stability Analysis of the South Baldy and Springerville Sites", MMA Memo 68.

2. Holdaway, M. 1992, "Possible Phase Calibration of the MMA", MMA Memo 84.

3. Holdaway, M. 1995, "A Test of Fast Switching Phase Calibration with the VLA at 22 GHz", MMA Memo ??.

4. Ishiguro, M., Kanzawa, T., and Kasuga, T., "Monitoring of Atmospheric Phase Fluctuations Using Geostationary Satellite Signals", IAU Symposium on *Radio Astronomical Seeing*, eds J.E. Baldwin & Wang Shouguan. IAP, Beijing.

5. Masson, C. 1994, "Atmospheric Effects and Calibration", in *Astronomy with Millimeter and Submillimeter Wave Interferometry*, eds Ishiguro and Welch.

6. Sramek, R. A. 1990, IAU Symposium on *Radio Astronomical Seeing*, eds J.E. Baldwin & Wang Shouguan. IAP, Beijing.

7. Treuhaft and Lanyi, 1987, "The Effect of the dynamic wet troposphere on radio interferometric measurements", Radio Science, Vol 22, No. 2, 251-265.

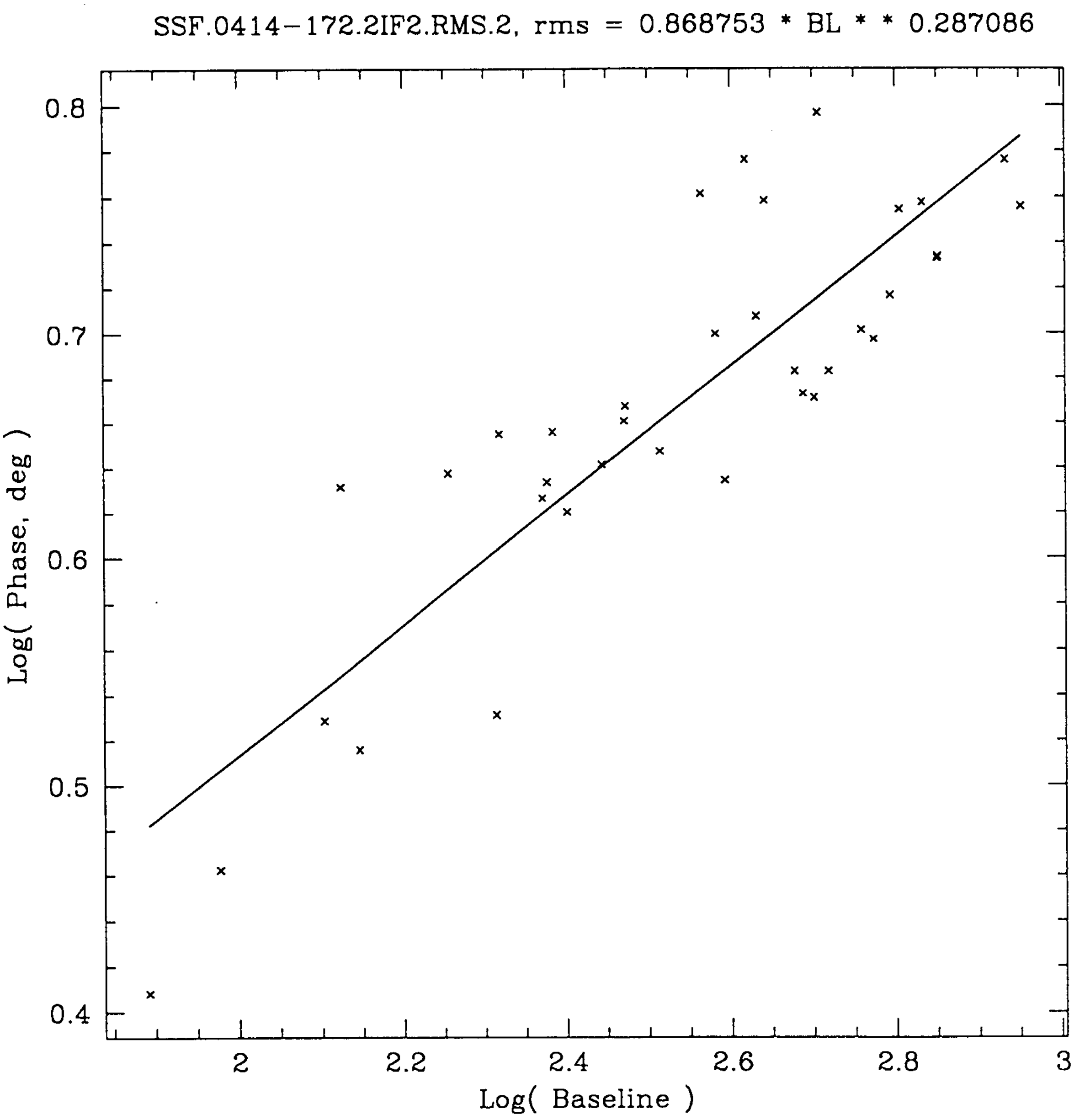

Figure 1: Rms phase fluctuations as a function of baseline length for the source 0454-226, IF 2, demonstrating the typical quality of fit for our observations.

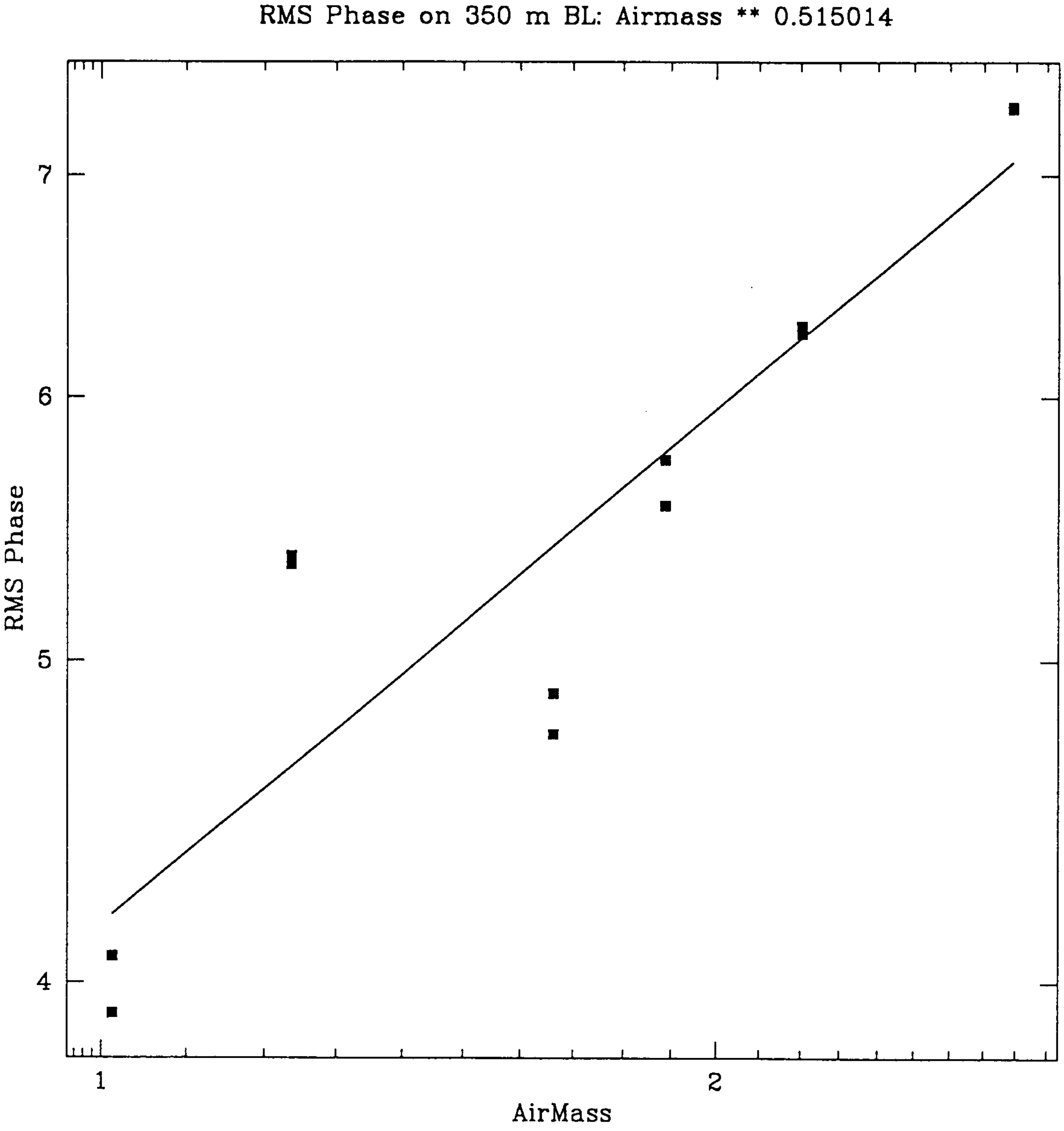

Figure 2: Rms phase fluctuation estimated for a baseline of 350 m as a function of airmass, displaying a relationship very close to $\sigma_\phi \propto (airmass)^{0.5}$.

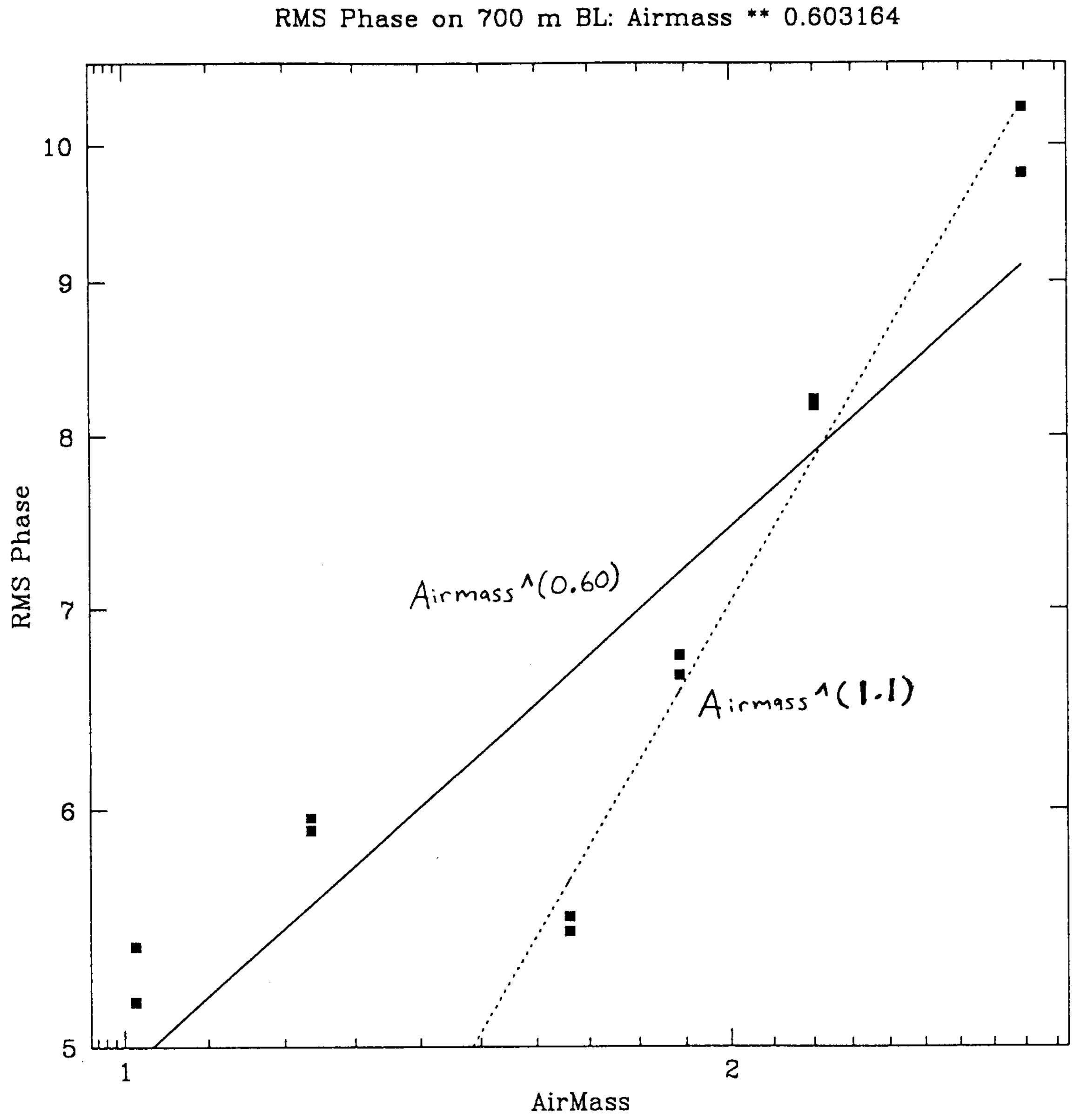

Figure 3: Rms phase fluctuation estimated for a baseline of 700 m as a function of airmass. If one includes all data points in the fit, the rms phase varies as airmass raised to the 0.60 power, which is probably not significantly different from the theoretical 0.50. However, if the two low airmass data points are not included in the fit, the rms phase varies as the airmass raised to the 1.1 power, which would indicate the scale height of the turbulent water vapor is less than 700 m.